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**SONIC REFLECTION FROM SOLID PLATES**

by

Walter G. Mayer

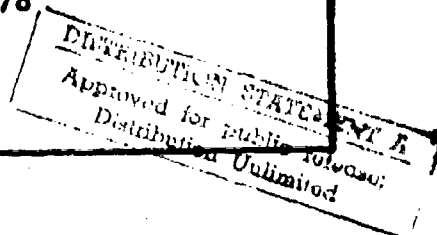
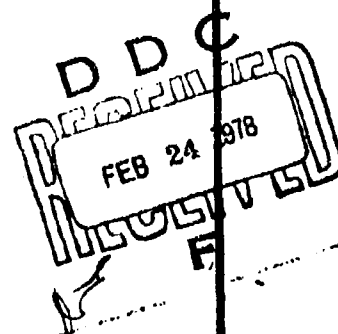
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Theoretical development for reflection effects for a bounded ultrasonic beam from flat interfaces and plates. Experimental observations for reflection from thick solid plates.		

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This Technical Report contains three publications which were prepared between 1 May 1977 and 31 December 1977:

- 1) a paper on sonic reflectivity which appeared in the September 1977 issue of the journal Ultrasonics,
- 2) a summary of a paper on nonlinear characteristics of plate modes which was presented at the 9th ICA in July 1977,
- 3) a paper on sonic reflections from thick solid plates which has been submitted to the Journal of Geophysical Research in November 1977.

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WALTER G. MAYER

Principal Investigator

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# Non-specular reflection from solid plates and half-spaces

L.E. PITTS and W.G. MAYER

Recent theoretical developments for the reflection effects seen for an ultrasonic bounded beam reflected from a solid surface are presented and a discussion of the implications of these developments is related to the case of reflection from infinite half-spaces and solid plates.

## Introduction

Ultrasonic beams have been used in the areas of ultrasonic testing and measurement for many years. However, a unified theoretical description of the reflection of ultrasonic beams from solid surfaces in a liquid medium is a relatively recent development.<sup>1-3</sup> Theoretical results presented<sup>1,3</sup> indicate that a new perspective is necessary when discussing the reflection effects from solid surfaces. In particular, the ramifications of using a thick solid plate to represent an infinite half-space remains somewhat clouded by the new experimental and theoretical results during the past several years. This paper is meant to be a descriptive commentary on the assumptions about the different effects encountered when using 'thick' solid plates as opposed to the effects seen from 'thin' solid plates for the reflection of an ultrasonic beam which is incident from a liquid medium.

## The beam reflection mechanism

A theoretical analysis of a reflected sound beam can be accomplished by employing Fourier transforms<sup>1,3</sup> to show that an incident bounded beam is mathematically equivalent to a sum of infinite plane waves of identical frequency but with different amplitudes and at different incident angles. Thus, the incident beam is a superposition of these infinite plane waves. Therefore, the reflected sound field, for a beam incident on a solid plate, is a sum over each of the constituent reflected infinite plane waves, where the amplitude and phase change of each plane wave is given by the plane wave amplitude reflection coefficient. The amplitude profile of the reflected sound field is determined by the superposition of the reflected infinite plane waves.

Since most ultrasonic transducers produce beams with an approximately Gaussian profile (spacial dependence of the

pressure), it will be assumed throughout that the incident sound beam is Gaussian. If the surface of reflection is taken as the x-y plane, and the beam is incident with a wave vector in the x-z plane, the incident particle displacement,  $U_i$  in normalized form, can be taken to be of the form

$$U_i(x, z) = \exp \{ -(x \cos \theta_i / W - z \sin \theta_i / W)^2 + i x k_i + i z k'_z - i \omega t \} \quad (1)$$

where  $\theta_i$  is the incident angle of the beam,  $W$  is one-half the beam width and  $k$  is the incident wave vector. Other variables in (1) are  $\omega$ , the angular frequency, and  $k'_z = \sqrt{(k^2 - k_i^2)}$ , where  $k_i = k \sin \theta_i$ . By employing the Fourier transform, an alternative relationship for the incident beam is

$$U_i(x, z) = (1/2\pi) \int_{-\infty}^{+\infty} V(k_x) \times \exp \{ i (x k_x + z k_z) - i \omega t \} dk_x \quad (2)$$

where  $k_z^2 = k^2 - k_x^2$  and

$$V(k_x) = \int_{-\infty}^{+\infty} U_i(x, 0) \exp \{ -i x k_x - i \omega t \} dx \quad (3)$$

Employing the definition of  $U_i$  in (1),  $V(k_x)$  becomes

$$V(k_x) = (\sqrt{\pi} W / \cos \theta_i) \times \exp \{ -(k_i - k_x)^2 (W/2 \cos \theta_i)^2 \} \quad (4)$$

Thus,  $V(k_x)$  can be interpreted as the amplitude of an incident infinite plane wave, incident from the angle  $\theta$ , where  $k_x = k \sin \theta$ .

Hence, the reflected sound profile can be defined by summing over all of the individual infinite plane waves by using the reflection coefficient of infinite plane waves,  $R(k_x)$ . Thus, the reflected sound field is given by

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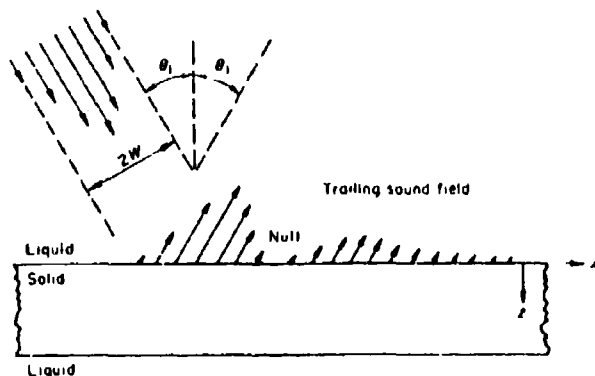


Fig. 1 Schematic diagram of typical non specular reflection from a solid plate in a liquid. The length of arrows is approximately proportional to sound intensities. (After Ref. 3)

$$U_R(x, z) = (1/2\pi) \int_{-\infty}^{+\infty} R(k_x) V(k_x) \times \exp \{ i(xk_x - zk_z) - i\omega t \} dk_x \quad (5)$$

At the surface of the interface,  $z = 0$ , the above equation reduces to

$$U_R(x, 0) = (1/2\pi) \int_{-\infty}^{+\infty} R(k_x) V(k_x) \times \exp \{ i x k_x - i\omega t \} dk_x \quad (6)$$

Thus, the reflected sound field can be found if the plane wave reflection coefficient is known.

It has been noted experimentally that an ultrasonic beam reflected from a solid interface undergoes a non-specular reflection when the beam is incident at certain angles. Fig. 1 shows schematically the central features of the reflected sound field for one of these non-specular reflections. The length of the arrows in the figure corresponds to the sound intensity levels. The reflected sound field has three significant features: (1) the presence of a null intensity point not found in the incident beam, (2) a lateral displacement of the main portion of the reflected beam along the interface of the boundary and (3) the presence of what will be called the 'trailing sound field'. The dashed line in the figure shows a specularly reflected beam.

The physical mechanism for the non-specular reflection can be analysed by considering the form of  $U_R(x, 0)$ , given by (6). The term  $V(k_x)$  acts as a damping factor in the integral such that the integrand is appreciably large only when  $V(k_x)$ , a Gaussian function given in (4), is large. Thus the integration is effectively completed over only a small region of  $k_x$  values when  $k_x \approx k_i$ . The other important factor in the integrand is the reflection coefficient,  $R(k_x)$ . It can be shown that for any set of solid and liquid materials, the reflection coefficient can be written in terms of only two variables:<sup>4</sup> the sine of the incident angle,  $\sin \theta$ , and the product of the frequency and the thickness of the plate,  $fd$ . Fig. 2 shows a typical reflection coefficient for a brass plate in water for which the  $fd$  parameter is 6 MHz mm. The non-

specular reflections occur when the Gaussian beam is incident such that  $\sin \theta_i$  (and thus the function  $V(k_x)$ ) is appreciably large) is near a point where the reflection coefficient undergoes a major change. As Fig. 2 indicates, there are two different types of major changes in  $R(k_x)$ . The first type occurs when the reflection coefficient amplitude undergoes a rapid decrease to zero while the phase of the reflection coefficient undergoes a phase change of  $2\pi$  radians. Thus, the infinite plane waves making up the incident beam undergo an amplitude and phase change. The superposition of the reflected infinite plane waves makes up the reflected sound field. The second type of major change in  $R(k_x)$  results at the point where the reflection coefficient amplitude remains constant and the phase of the reflection coefficient changes by approximately  $2\pi$ . For reasons to be explained later, the first type of change will be referred to as a Lamb-type change and the second type will be called a Rayleigh-type change. Hence, the important aspect of the reflection coefficient is that changes in the reflection profile are caused by the interference of the reflected infinite plane waves making up the incident beam, each of which is reflected with a different amplitude and phase shift.

The integral in (6) has been evaluated<sup>3</sup> by using a contour integration in complex  $k_x$  space and the residue theorem. Thus, the solution can be well approximated by utilizing the poles and zeroes of the function  $R(k_x)$  in the complex  $k_x$  plane. An analysis of the reflection coefficient has shown<sup>1,4</sup> that the zeroes and poles exist in pairs, and the types of poles can be categorized in terms of the relationship of the zero to the pole in the complex  $k_x$  plane. One type of pole zero pair has the pole in the complex plane and the zero on the real  $k_x$  axis, thus causing the zeroes of  $R(k_x)$  seen in Fig. 2. Since the real part of the pole location is related to a Lamb-mode of vibration of the plate,<sup>4</sup> this pole will be called a Lamb-type pole and the non-specular reflection caused by the zero/pole pair will be called a Lamb-type reflection. The second type of zero/pole pair has a pole in the complex  $k_x$  plane and a zero at the mirror image across the real  $k_x$  axis. The real part of this pole is related to a Rayleigh-mode of vibration of the plate. Thus, it will be called a Rayleigh-type pole. The major changes of the reflection coefficient outlined earlier can be directly related to the zero/pole pairs. It should be noted that Fig. 2 indicates

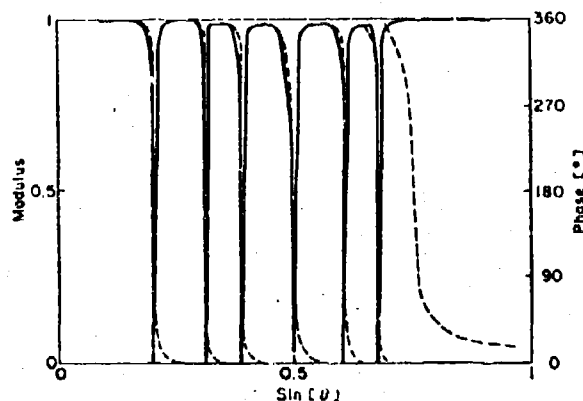


Fig. 2 Modulus (solid lines) and phase (dashed lines) of reflection coefficient as a function of  $\sin \theta$  for a brass plate in water with an  $fd$  of 6 MHz mm

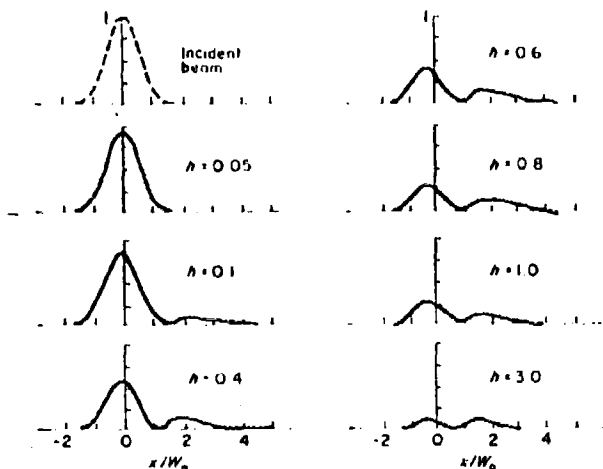


Fig. 3 Lamb-type reflections of sound intensity for various values of the parameter  $h$ . (After Ref. 3)

a major distinction between the types of non-specular reflection effects which one expects to see. For Lamb-type reflections, the amplitudes of the infinite plane waves decrease upon reflection, due to transmission through the plate. However, for Rayleigh-type reflections, the energy is totally reflected, although the beam loses its previous definition and results in a non-specular reflection profile.

Theoretical analyses of the reflection profile of a Gaussian incident beam from a liquid onto a solid infinite half-space<sup>1</sup> (L/S interface) and for a beam incident on a solid plate<sup>3</sup> (L/S/L interface) have been accomplished. For the L/S interface, it has been found that the reflection coefficient has only one significant pole/zero pair and it is a Rayleigh-type pole. However, for the L/S/L interface case (plate), there exists one Rayleigh-type pole and several Lamb-type poles, as indicated by  $R(k_x)$  in Fig. 2. In each case, the reflected sound field for  $k_l = \text{Re}(k_p)$  where  $k_p$  is a pole location of  $R(k_x)$ , can be parameterized in terms of a single parameter,  $h$ , which has the following form:

$$h = \frac{2\pi W f \text{Im}(\sin \theta_p)}{(V \cos \theta_l)} \quad (7)$$

$$= W \text{Im}(k_p) / \cos \theta_l$$

where  $k_p$  is the pole location (corresponding to the  $\sin \theta$  value in Fig. 2 where the reflection coefficient undergoes a major change),  $k_p = k \sin \theta_p = 2\pi f \sin \theta_p / V$ , where  $V$  is the velocity of the sound in the liquid. Since the reflection coefficient undergoes different types of changes for the two types of poles, one expects different types of beam profiles (Lamb-type reflections and Rayleigh-type reflections). In terms of the parameter  $h$ , the non-specular reflection effects are shown in Figs 3 and 4 for several values of  $h$ .

The primary difference between the two types of non-specular reflections is that all of the incident sound energy is reflected for the Rayleigh-type reflection, while a transmitted sound field which is present for the Lamb-type reflection causes a decrease in the total reflected energy as  $h$  increases. It can be seen from Figs 3 and 4 that non-specular reflection effects occur for a range of  $h$  values from  $h = 0.05$  to approximately  $h = 4$ .

## Experimental considerations and the $h$ parameter

It was shown in the previous section that a non-specular reflection effect can be completely described by a single parameter  $h$ . An analysis of this parameter will provide information about what physical situations will result in non-specular reflection effects. It is important to realize that the knowledge of  $k_l = \text{Re}(k_p)$  is not sufficient information to predict a non-specular reflection, since the  $h$  parameter must be within the limits of  $0.05 < h < 4$ . Clearly, a prediction of a non-specular reflection involves a knowledge of both real and imaginary parts of the function  $\sin \theta_p$  where  $\sin \theta_p = k_p/k$ . An analysis of the pole locations of the reflected coefficient<sup>4</sup> shows that the pole location is a function of the parameter  $fd$  where  $f$  is the frequency and  $d$  is the thickness of the solid plate. Fig. 5 shows a typical case for the location of the real parts of the pole locations as a function of  $fd$  for a brass plate in water. The variable used is  $\text{Re}(\sin \theta_p)$ . The pole trajectories as a function of  $fd$  are labelled as symmetric or antisymmetric modes, since each pole is related to a possible mode of vibration of the

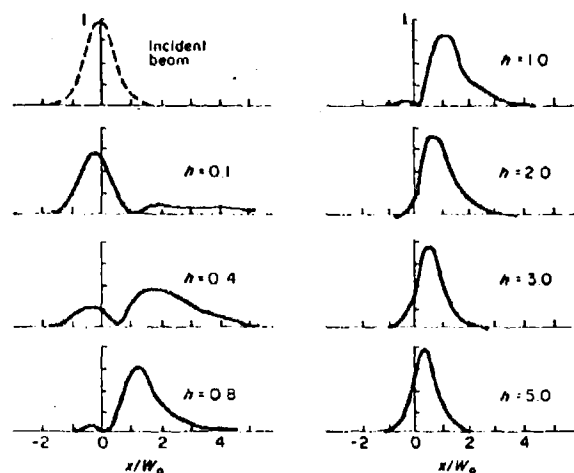


Fig. 4 Rayleigh-type reflection of sound intensity for various values of the parameter  $h$ , calculated after Ref. 1

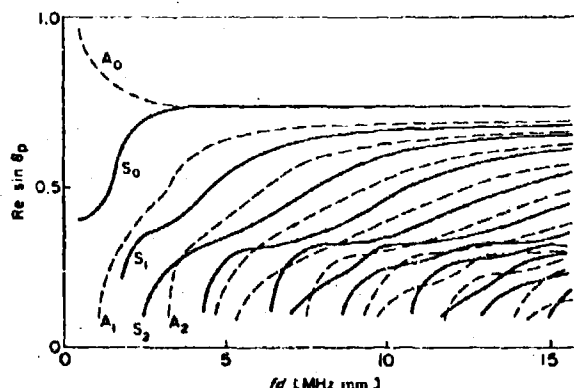


Fig. 5 Real part of  $\sin \theta_p$  as a function of  $fd$  for a brass plate in water. (After Ref. 4)

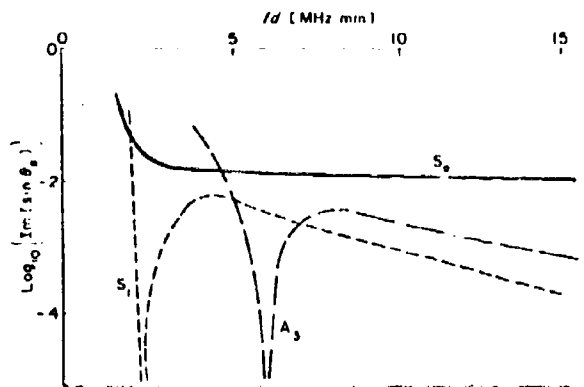


Fig. 6 Imaginary part of  $\sin \theta_p$  for some modes of vibration of a brass plate in water as a function of  $fd$  (After Ref. 4)

solid plate. Fig. 6 shows the imaginary part of the pole locations for three poles as a function of  $fd$  for a brass plate in water.

As  $fd$  increases (increasing plate thickness) it can be seen that a definite difference develops between the zeroth order poles (Rayleigh-mode pole) and the other pole trajectories, in that for the Rayleigh-mode poles, the imaginary part of the pole location is constant as a function of  $fd$ , while for the Lamb-mode poles, the imaginary part of the pole location decreases for increasing values of  $fd$ . Most experimental work on Lamb-type reflections has been done at relatively small  $fd$  values ( $fd$  less than 12 MHz mm). However, the results shown here indicate that the non-specular reflection effects depend only on the parameter  $h$ . Thus, even for large  $fd$  values, a non-specular reflection effect could be possible if  $h$  were greater than 0.05. If  $\text{Im}(\sin \theta_p)$  decreases with increasing  $fd$ , the value of  $f$  must be large to see a non-specular reflection effect. Hence Lamb-type non-specular reflection effects are generally observable for large  $fd$  values when high frequencies and thin plates are used. Thus, the use of thin plates has been associated with the presence of Lamb-type non-specular reflections, although the exact definition of 'thin' remains ambiguous.

A second aspect of the analysis of reflections from an L/S/L interface involves the connection between the L/S and L/S/L interface cases. As mentioned previously, when an analysis of the L/S (half-space) reflection is undertaken,<sup>1,3</sup> it is found that only one pole, the Rayleigh pole, is found.

Thus, an infinite half-space would produce no Lamb-type reflections. Since the Lamb-type reflections are governed by the parameter  $h$ , if  $h$  is very small, no Lamb-type reflections will be seen. However, from the fact that  $\text{Im}(\sin \theta_p)$  generally decreases as  $fd$  increases it should not be erroneously interpreted that  $h$  can be assumed small, because for large  $f$  values,  $h$  could be appreciably large even though  $fd$  is large. A recent study<sup>5</sup> has shown that Lamb-type reflections can be seen at  $fd$  values of at least 35 MHz mm when the plate thickness could generally be considered large. For example, for a glass plate in water, for  $fd = 30$  MHz mm, there are Lamb-type poles with  $\text{Im}(\sin \theta_p)$  values of approximately 0.002. For  $f = 3$  MHz and  $d = 10$  mm (a thick plate), this would result in an  $h$  value of 0.24 when  $W = 9.5$  mm. Although this plate thickness would generally be considered to be sufficient to assume that the plate would act as an infinite half-space, it is clear that Lamb-type reflections are present. Therefore, it is seen that the use of a 'thick' solid plate to approximate an infinite half-space is not sufficient for excluding Lamb-type reflections.

It should be noted that the pole locations for most metals which have been considered show that  $\text{Im}(\sin \theta_p)$  for  $fd$  greater than 20 MHz mm is less than 0.0008. Therefore, for most metals, when frequencies under 10 MHz are used,  $h$  will be less than 0.05 so that no Lamb-type reflections will be observed. Since  $\text{Im}(\sin \theta_p)$  corresponding to a Rayleigh-type pole is constant as  $fd$  increases, the single Rayleigh-type non-specular reflection will be seen. Thus, under certain conditions, large  $fd$  values do allow a 'thick' plate to be used as an approximation to an infinite half-space, although a prior knowledge about the locations of the poles of the reflection coefficient is necessary to make the prediction of the accuracy of the assumptions used.

### Acknowledgement

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## NONLINEARITIES IN LAMB WAVES

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## INTRODUCTION

The equations of motion become nonlinear for finite amplitude Lamb modes propagating in isotropic plates. Hence, mode interactions may occur. Two such interactions are the Lamb mode three-phonon interaction and Lamb mode second harmonic generation. These two interactions are investigated experimentally.

## THEORETICAL BASES

The nonlinear equations of motion for an isotropic plate yield criteria for and characteristics of the generated waves. Specifically, sufficient conditions for generating third phonons are

$$\omega_3 = \omega_1 + \omega_2 \quad \text{and} \quad \bar{k}_3 = \bar{k}_1 + \bar{k}_2, \quad (1)$$

where  $\omega$  and  $\bar{k}$  are the angular frequencies and wave vectors of the Lamb modes. The generated third mode has an amplitude

$$A_3 = A_0 B_0 \quad (2)$$

where  $A_0$  and  $B_0$  are the amplitudes of the generating (pump) modes.

A necessary condition for Lamb mode second harmonic generation is

$$V_2 = V_1 \quad (3)$$

where  $V_1$  and  $V_2$  are the velocities of the fundamental and the second harmonic Lamb mode. A generated second harmonic has an amplitude

$$A_2 = (A_1)^2 x \quad (4)$$

where  $A_1$  is the amplitude of the fundamental and  $x$  is the propagation distance.

## EXPERIMENTAL TECHNIQUES

The criteria for generation, eqs. 1 and 3, are used to set up the pump modes for the two interactions. The means of detection of possible generated modes is an optical probe where a laser beam interacts with plate modes to yield a light reflection/diffraction pattern (1). The velocity and frequency of the constituent Lamb modes may be obtained from this pattern. If harmonics are present, the diffraction pattern is asymmetric (2). Further, if a third phonon is generated, then a third diffraction pattern is present.

## RESULTS

In the second harmonic experiment, an asymmetric light diffraction pattern is observed. Further, the asymmetry increases with propagation distance,  $x$ . This effect is predicted by eq. 4 for a generated Lamb second harmonic. Thus harmonic generation in Lamb modes is observed.

In the three-phonon interaction experiment, a third diffraction pattern is detected. The Lamb mode which creates the pattern has a velocity and frequency which corresponds to the calculated values for a third phonon. Further, the amplitude of this Lamb mode, as given by eq. 2, is measured. Hence the three-phonon interaction is observed for plate modes.

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## Nonspecular Ultrasonic Reflection From Thick Solid Plates

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### ABSTRACT

Nonspecular reflection of an ultrasonic bounded beam from thick plates in water are observed. It is shown that the reflected beam profile does not only depend on the product frequency of the ultrasound times plate thickness, among other parameters, but also on the frequency alone. Furthermore, results indicate that a thick plate may not be a valid approximation of an infinite halfspace as has been commonly assumed.

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## INTRODUCTION

When a bounded ultrasonic beam is incident onto a solid plate immersed in a liquid there are a number of angles of incidence, called the Lamb angles,  $\theta_L$ , at which nonspecular reflected beam effects are observed.<sup>1</sup> An ultrasonic beam incident at these angles satisfies the conditions under which vibrational modes of the plate (Lamb modes) are excited. These nonspecular effects at such a liquid/solid-plate/liquid interface (L/S/L) are similar to the ones observed at a liquid/solid (L/S) interface, where the reflected beam may become much wider than the incident beam or may have an intensity profile quite different from that of the incident beam<sup>2-5</sup>. These nonspecular reflections occur on an L/S interface at only one angle of incidence, the Rayleigh angle,  $\theta_R$ . Both the L/S<sup>6</sup> and L/S/L<sup>7</sup> cases have been treated theoretically.

Experimental observation of nonspecular effects of the L/S/L case have been confined to a small range of values of  $fd$  (the product of frequency of the ultrasonic beam and plate thickness),  $\sim 1 < fd < 14$  (MHz.mm). It had been generally accepted that as one goes to higher values of  $fd$ , it is no longer possible to observe all of the predicted vibrational modes of the plate. In this case  $d$  becomes large enough so that in the limit  $L/S/L \rightarrow L/S$  and this infinite halfspace interface supports only one mode of vibration - the Rayleigh mode.

However, the theoretical investigation of the L/S/L case, predicts that for incidence at the Lamb angle,  $\theta_L$ , the profile of the reflected beam depends not only on  $fd$  but also on  $f$  alone.

The purpose of this paper is to give experimental evidence that: (1) the nonspecular reflected beam effects are present at larger values of  $fd$  than had

been observed before, and (2) the reflected beam profile does indeed depend on the frequency of the incident ultrasound.

#### THEORETICAL CONSIDERATIONS

Nonspecular reflection effects of L/S/L interface can be described in terms of poles and zeroes of the infinite plane wave amplitude reflection coefficient<sup>7</sup>. The real part of the pole location is related to the angles of incidence at which Lamb waves are generated on the plate,  $\theta_L = \text{Re}(\theta_p)$ . These angles can be expressed as a function of the product  $fd$ .

The imaginary part of the pole location,  $\text{Im} \sin \theta_p$ , is related to a parameter  $h$  defined as

$$h = (\text{Im} \sin \theta_p) (2\pi f W_0 / V),$$

where  $W_0 = W/\cos \theta_L$  and  $W$  is half of the incident beam width;  $V$  is the sonic velocity in the liquid. The parameter  $h$  determines the profile of the reflected beam and hence can be used to predict whether or not nonspecular effects are observable<sup>1</sup>.

The nonspecular reflected beam effects are readily observable for a certain range of the  $h$  parameter,  $0.1 \lesssim h \lesssim 3$ . For most solids the value of  $\text{Im} \sin \theta_p$  is very small and therefore wide beams and/or high frequencies are needed to have an  $h$  value which is within this range. Calculations show that for glass the value of  $\text{Im} \sin \theta_p$  is rather large that even for a relatively narrow beam (19 mm width) and frequencies of the order of a MHz, the value of  $h$  is large enough to render the nonspecular effects observable. This has led to the selection of glass as the material to be investigated in this paper.

Due to the dependence of the reflected beam profile on the parameter  $h$ , one notes that for a particular Lamb mode of the plate, i.e. for a given  $fd$

and  $\theta_L$ , one can change the  $h$  parameter (and thus the reflected beam profile) by changing  $t$ . If this change in  $f$  is accompanied by the appropriate change in  $d$ , the product  $fd$  will remain unchanged. It should be noted that the value of  $\sin \theta_p$  remains constant for a constant  $fd$  at a particular Lamb mode.

It is often assumed that for large values of  $fd$  the L/S/L case is essentially identical to the L/S halfspace. In fact in all of the experimental results dealing with the L/S case, a thick plate has been used as a valid approximation of an infinite halfspace. However, observe that even for a 15 mm thick plate one cannot assume that the only mode of vibration is the Rayleigh mode, indicating that the halfspace approximation may not be reliable.

#### EXPERIMENT AND RESULTS

The experiment is done using 3x3 inch square samples of glass of different thicknesses. The shear and longitudinal velocities of the glass used are 3445 m/sec and 5775 m/sec respectively. The density of the samples is 2.49 gr/cm<sup>3</sup>. The samples are immersed in water and quartz transducers are used to generate bounded ultrasonic beams of 19 mm width at frequencies of 2.9 and 16 MHz.

Schlieren techniques are used to observe and photograph the reflected beam profiles. Nonspecular effects are observed when the angle of incidence corresponds to one of the Lamb angles,  $\theta_L$ , of the glass plate.

Figures 1 and 2 are representative Schlieren photographs of the nonspecular reflected beam effects that are observed when  $fd = 35.5$  MHz.mm. Each figure represents a particular vibrational mode of the glass plate for a fixed  $fd$ , but for different Lamb angles,  $\theta_L$ . Part (a) in each figure corresponds to a frequency of 2.9 MHz while part (b) corresponds to a frequency of 16 MHz.

The results clearly show that the nonspecular reflected beam effects are

present even for the relatively high value of  $fd$ . Comparing parts (a) and (b) in each figure, for the particular Lamb mode that it represents, one notices that the profile of the reflected beam changes markedly with frequency as predicted by the theory.

The theoretical description of the beam profile contains<sup>8</sup> considerations of specific poles of the reflection coefficient. Theory predicts that at high frequencies, 16 MHz as used in this experiment, the reflected beam profile is affected by a single pole, whereas at the low frequency of 2.9 MHz more than one pole contributes to the reflected beam profile. Since the existing theory is based on an  $h$ -parameter defined for a single pole contribution, one notes that the predictions of the reflected beam profile, as provided by the value of the  $h$ -parameter, is valid only when the contribution comes from a single pole, as in part (b) of each figure. In the case of low frequencies more than one pole contributes, and the multiple intensity peaks and the wide trailing field of the part (a) in each figure cannot be explained by the  $h$ -parameter. These facts are in agreement with the experimental results that are observed.

#### CONCLUSION

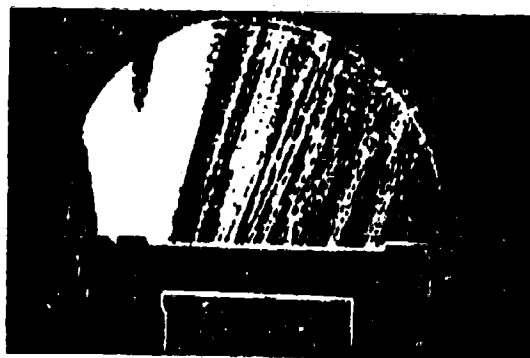
Nonspecular reflected beam effects are observed from a glass plate immersed in water at an  $fd = 35.5 \text{ MHz.mm}$ , a result indicating that even for this high value of  $fd$  one does not approach the configuration of an L/S halfspace, and that the Lamb modes of vibration are still present and observable.

For a given vibrational mode of the plate material the intensity profile of the reflected beam does depend on the frequency of the incident sound as predicted by theory.

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a



b



a



b

Fig. 2



## FIGURE CAPTIONS

Figure 1. Schlieren photographs of the nonspecular reflected beam effects at water/glass-plate/water interface for  $fd = 35.5 \text{ MHz}\cdot\text{mm}$ .  
(a)  $f = 2.9 \text{ MHz}$ ,  $d = 12.25 \text{ mm}$ . (b)  $f = 16 \text{ MHz}$ ,  $d = 2.22 \text{ mm}$ .  
Sound is incident from the upper left side of the picture.

Figure 2. Schlieren photographs of the nonspecular reflected beam effects at water/glass-plate/water interface for  $fd = 35.5 \text{ MHz}\cdot\text{mm}$ .  
(a)  $f = 2.9 \text{ MHz}$ ,  $d = 12.25 \text{ mm}$ . (b)  $f = 16 \text{ MHz}$ ,  $d = 2.22 \text{ mm}$ .  
Sound is incident from the upper left side of the picture.